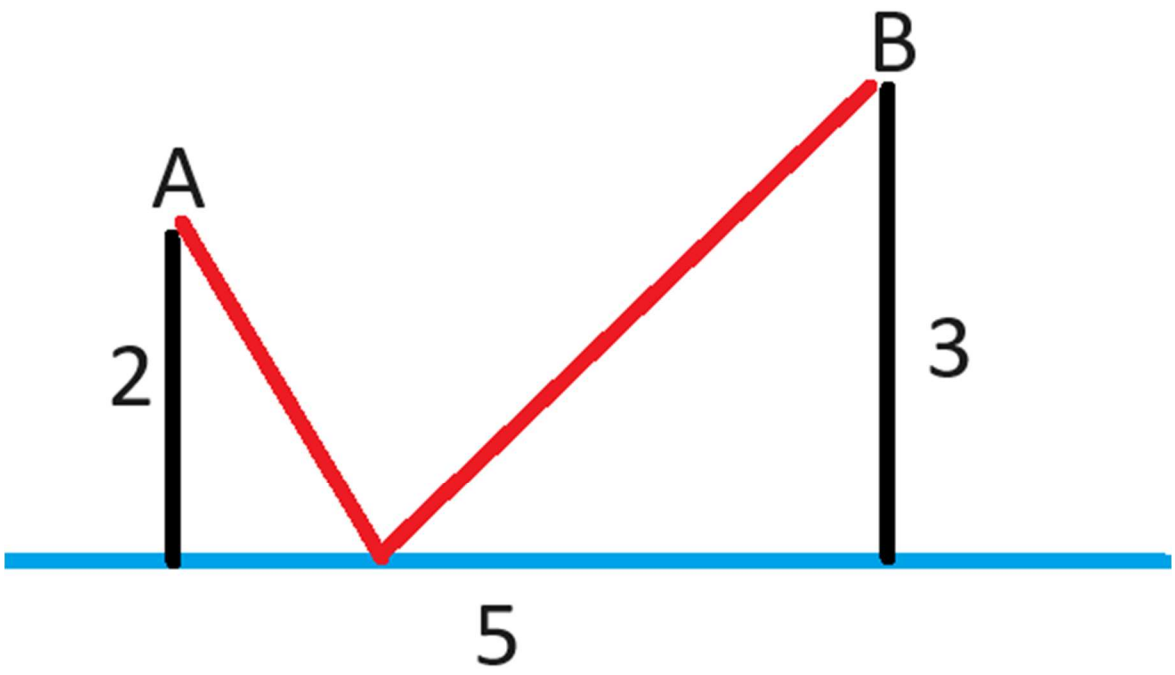
Question

There is a straight water pipe (blue) in the vicinity of points A and B. Point A is 2 miles from the closest point on the pipe. Point B is 3 miles from the closest point on the pipe. These two points along the pipe that mark the closest points to A and B are 5 miles apart. It is desired to lay two new pipes (red), linking A and B to the water-bearing pipe, having only one point of contact with the water pipe with the two new pipes going directly to A and B. In other words, the new pipes must form a V shape. What is the least distance of pipe required?



Answer

$5\sqrt{2}$ =~ 7.071068 miles

Hard Solution

Let C = closest point on the blue pipe to point A.

Let D = closest point on the blue pipe to point B.

Let E = point on the blue pipe to connect the red pipes to A and B.

Let x = Distance from C to E.

Using the Pythagorean formula, the distance from A to E is:

$$\sqrt{x^2 + 4}$$

Using the Pythagorean formula, the distance from B to E is:

$$\sqrt{x^2 - 10x + 34}$$

Let f(x) = Total length of red pipes =

$$f(x) = \sqrt{x^2 + 4} + \sqrt{x^2 - 10x + 34}$$

To find the minimum value of x, take the derivative of f(x), set it equal to zero, and solve for x.

$$f'(x) = \frac{(1/2) * 2x}{\sqrt{x^2+4}} + \frac{2x-10}{\sqrt{x^2-10x+34}} = 0$$

$$x\sqrt{x^2 - 10x + 34} = (5-x)\sqrt{x^2 + 4}$$

Square both sides:

$$x^2\,(x^2 - 10x + 34) = (5 - x)^2\,(x^2 + 4)$$

$$x^4 - 10\,x^3 + 34x^2 = \; x^3 - 10x^3 + 29x^2 - 40x + 100$$

The x^4 – 10 x^3 conveniently cancel from both sides:

$$34x^2 = \; 29x^2 - 40x + 100$$

$$5x^2 + 40x - 100 = 0$$

$$x^2 + 8x - 20 = 0$$

$$(x + 10)\,(x - 2) = 0$$

$$X = +2, -10$$

-10 is clearly not the answer, thus the red pipes should connect 2 miles from point C.

That makes the total distance of the red pipes:

$$\sqrt{2^2 + 4} + \sqrt{2^2 - 20 + 34}$$

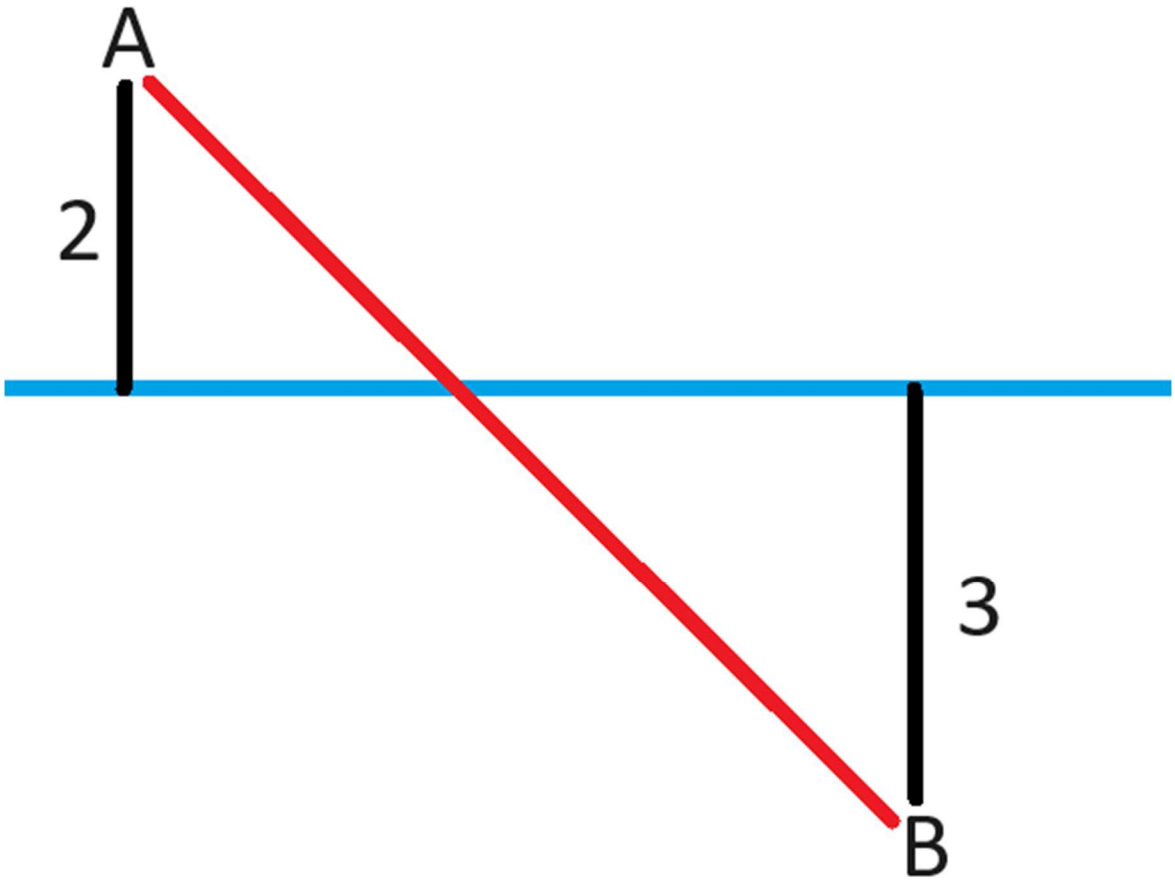
$$= \sqrt{8} + \sqrt{18}$$

$$= 2\sqrt{2} + 3\sqrt{2}$$

$$= 5\sqrt{2} = \sim 7.071068$$

Easy Solution

It stands to reason the answer would be the same if point B were on the opposite side of the blue pipe, all other things being the same.



The solution would then simply be linking the red pipe directly from A to B. A and B are 5 miles apart vertically and horizontally. By the Pythagorean formula, the red pipe would have distance:

$$\sqrt{5^2 + 5^2} = \sqrt{50} = 5\sqrt{2} =\sim 7.071068$$