Question

You receive an average of one Email every six minutes. The probability of receiving an Email any given moment is constant and independent of the time since the last Email was received. Every minute you roll a six-sided die, starting one minute from now. What is the probability you receive an Email before rolling a six?

Answer

$$\frac{1-e^{-1/6}}{1-\frac{5}{6}e^{-\frac{1}{6}}} =\sim 0.521110061634751$$

Solution

Let p = probability Email received first.

The probability of not receiving an Email in one minute is $e^{-\frac{1}{6}} = \sim 0.846482.$

So, the probability of receiving at least one Email in one minute is $1\text{-}e^{-\frac{1}{6}} = \sim 0.153518.$

After one minute, if no Email is received, there the probability of not rolling a six is 5/6. Assuming both evenings happen (no Email in first minute and surviving one roll of the die), then you're right back where you started at the beginning, with the same chance of receiving an Email first. We can express this mathematically as:

$$p = (1\text{-}e^{-1/6}) + e^{-1/6}\,\frac{5}{6}\,p$$

Solve for p:

$$p\,(1\text{-}e^{-1/6}\,\frac{5}{6}) = (1\text{-}e^{-1/6})$$

$$p = \frac{\mathbf{1}-\boldsymbol{e}^{-\mathbf{1/6}}}{\mathbf{1}-\frac{\mathbf{5}}{\mathbf{6}}\,\boldsymbol{e}^{-\mathbf{1/6}}}$$