Question

Your email account receives an average of one email every six minutes. The probability of receiving an Email at any given moment is always the same and not correlated with the time since the last Email was received. Once a minute, starting one minute from now, you roll a die until you roll a 1. What is the expected waiting time until the first thing to happen between receiving an Email or rolling a 1?

Answer

The answer is $\frac{6\,(1-e^{-1/6})}{1-\left(\frac{5}{6}\right)e^{-1/6}}$ =~ 3.126660 minutes.

Solution

The time between Emails follows an exponential distribution with a mean of 6.

The probability an Email has not been received by time t is given by $e^{-t/6}$. The probability an Email has not been received in the first minute is $e^{-1/6}$ =~ 0.846482.

We can calculate the expected waiting time for anything as the sum over all time that the event has not happened yet. For events that can happen at any moment, we can use integral calculus to find the answer. In the case of an Email, it is:

$\int_0^\infty e^{-t/6}\, dt$ = 6

The expected waiting time in the first minute is:

$\int_0^1 e^{-t/6}\, dt$ = 6 (1 - $e^{-1/6}$) =~ 0.921110.

Let's let a be the answer we're seeking, the expected total wait time until the first event.

a = expected wait time in first minute + probability no Email in first minute × (5/6) × a

What this is saying that if a minute goes by with no Email and no 1 rolled, then we're no closer to either event happening. Let's solve for a:

$a = 0.921110 + 0.846482 \times (5/6) \times a$

$a\,(1 - 0.846482 \times (5/6)) = 0.921110$

$a = 0.921110 / (1 - 0.846482 \times (5/6))$

$a = 0.921110 / 0.294599$

$a = 3.126660$

To put that in exact notation:

$$a = \frac{6\,(1 - e^{-1/6})}{1-\left(\frac{5}{6}\right)e^{-1/6}}$$