Q: Let $0<a<b$. What is more a^b or b^a ?

A: Let's take both terms to the power of 1/ab. That makes the question:

What is more $(a^b)^{1/ab}$ or $(b^a)^{1/ab}$? =

What is more $(a)^{1/a}$ or $(b)^{1/b}$?

Let's consider the question: If $y=(x)^{1/x}$, then if x increases slightly, then does $(x)^{1/x}$ increase or decrease?

To answer that, we must find dy/dx.

I don't like that 1/x in the exponent, so let's take the natural log of both sides to bring it down:

$\ln(y) = \ln((x)^{1/x})$

$\ln(y) = (1/x)*\ln(x)$

Now let's take the derivative of each side.

$1/y * dy/dx = -1/x^2 * \ln(x) + (1/x)*(1/x)$, via the Chain Rule.

$dy/dx = y * (-1/x^2 * \ln(x) + 1/x^2)$

$dy/dx = y * 1/x^2 * (1 - \ln(x))$

$dy/dx = (x)^{1/x} * 1/x^2 * (1 - \ln(x))$

Let's figure out when this function is increasing and decreasing.

If $1-\ln(x) =0$, then whole function must be zero, implying that $(x)^{1/x}$ does not change with a minute change in x.

Let's add ln(x) to both sides:

$1 = \ln(x)$

$x = e$.

So $y=(x)^{1/x}$ reaches either a maximum or minimum at x=e. We must find which it is.

Let's let x=3. Is dy/dx positive or negative at x=3?

$(3)^{1/3} * 1/3^2 * (1 - \ln(3))$

The first two parts of this product are obviously both positive. Ln(3) > 1, so (1-ln(3)) < 0.

Positive * positive * negative = negative.

So, if x>e, then dy/dx < 0.

Likewise, if x<e, then dy/dx > 0.

An earlier question was which is more $(a)^{1/a}$ or $(b)^{1/b}$, given b>a?

If b<e, then $(a)^{1/a} < (b)^{1/b}$, because $(b)^{1/b}$ is an increasing function in this range.

So, if $(a)^{1/a} < (b)^{1/b}$, then $a^b < b^a$.

Likewise, if a>e, then $(a)^{1/a} > (b)^{1/b}$, because $(b)^{1/b}$ is a decreasing function in this range.

So, if $(a)^{1/a} > (b)^{1/b}$, then $a^b > b^a$.

In simple terms, if both a and b are greater than e, then the expression with the greater exponent is greater. For example, 3^4 > 4^3, which is easy to verify: 81 > 64.

If both terms are less than e, then the expression with the smaller exponent is more. For example 2.5^2 > 2^2.5, which is easy to verify: 6.25 > 5.6569.

Where it's not so simple is if a<e<b.

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