

Let:

a = Rate Alice paints by herself.

b = Rate Betty paints by herself.

c = Rate Charlie paints by himself.

Total painting = rate * time.

In simple English, Alice and Betty accomplish in three hours what it takes Alice and Charlie to do in four hours and what it takes Betty and Charlie to do in five hours.

A way to express that numerically is:

$$3(a+b) = 4(a+c) = 5(b+c)$$

Let's try to solve for a and b in terms of c .

$$3a + 3b = 4a + 4c$$

$$(1) a = 3b - 4c$$

$$3a + 3b = 5b + 5c$$

$$2b = 3a - 5c$$

$$(2) b = (3a-5c)/2$$

Let's put the expression for b in (2) into equation (1):

$$a = 3(3a-5c)/2 - 4c$$

$$2a = 9a - 15c - 8c$$

$$7a = 23c$$

$$(3) a = (23/7)c$$

Let's put the value of a in (3) into equation (2):

$$b = (3 \cdot (23/7)c - 5c) / 2$$

$$2b = 3 \cdot (23/7)c - 5c$$

$$14b = 3 \cdot 23c - 35c$$

$$14b = 34c$$

$$(4) 7b = 17c$$

We can let c be anything we want, since the size of the house isn't specified. The two equations we're at now are:

$$7a = 23c$$

$$7b = 17c$$

Let's let $c = 7$

Then:

$$a = (23 \cdot 7) / 7 = 23$$

$$b = (17 \cdot 7) / 7 = 17$$

Let's make sure that's right. Recall:

$$3(a+b) = 4(a+c) = 5(b+c)$$

So:

$$3 \cdot (23+17) = 4 \cdot (23+7) = 5 \cdot (17+7)$$

$$3 \cdot 40 = 4 \cdot 30 = 5 \cdot 24$$

$$120 = 120 = 120$$

The house has an area of 120 to be painted. Let t = time it takes for all three to paint the house.

$$120 = t \cdot (23 + 17 + 7)$$

$$120 = 47t$$

$$t = 120/47 \approx 2.53191 \text{ hours.}$$