Question:

A grenade is thrown down a bottomless pit. On average, the grenade explodes in six seconds. The time until it explodes has a memoryless property, in that the probability of an explosion at any given moment is always the same and independent of low long it has been since the pin was pulled. In other words, it's lifetime follows the exponential distribution. Assume acceleration of 32 feet per second.

What is the mean distance the grenade falls before exploding?

Answer:

1152 feet.

Solution:

In general, random variable that follow the exponential distribution have a density function of :

f(t) = $B\ e^{-Bt}$

Where the mean lifetime is 1/B.

In this case, the density function is:

f(t) = (1/6) $e^{-t/6}$

The acceleration of the grenade is always 32.

It should be common knowledge that velocity is integral of acceleration. Thus, the velocity at time t is 32t. "What about the constant of integration?", you might ask. Since the velocity is obviously 0 at t=0, the constant of integration is conveniently 0.

It should also be common knowledge that the integral of velocity is distance traveled. So, if g'(t) = 32t, then g(t) = 16t^2. "What about the constant of integration?", you might ask again. Since the distance traveled at t=0 is obviously 0, that constant is, again, 0.

To get at the answer, we integration from t = 0 to ∞ of the product of the distance fallen at time t and the probability it explodes at that time. In other words:

$$\int_0^{\infty} 16t^2 \times e^{-t/6} \times \frac{1}{6}\, dt =$$

$$\frac{8}{3}\int_0^{\infty} 16t^2 \times e^{-t/6}\, dt$$

Next, do integration by parts:

u = t^2

du = 2t

v = $-6e^{-t/6}$

dv = $e^{-t/6}$

$$\frac{8}{3}\int_0^{\infty} 16t^2\ e^{-t/6}\, dt =$$

$$\frac{8}{3}\left[-6t^2 e^{-\frac{t}{6}} - \int_0^{\infty} -12t\ e^{-\frac{t}{6}}\right] dt =$$

$$\frac{8}{3}\left[-6t^2 e^{-\frac{t}{6}} + 12\int_0^{\infty} t\ e^{-\frac{t}{6}}\right] dt =$$

Next, do integration by parts again:

u = t

du = 1

v = $-6e^{-t/6}$

dv = $e^{-t/6}$

$$\frac{8}{3}\left[-6t^2e^{-\frac{t}{6}} + 12\int_0^{\infty} t\ \ e^{-\frac{t}{6}}\right] dt =$$

$$\frac{8}{3}\left[-6t^2e^{-t/6} + 12\left[-6te^{-\frac{t}{6}} - \int_0^{\infty} -6\ \ e^{-\frac{t}{6}}\right]\right] dt =$$

$$\frac{8}{3}\left[-6t^2e^{-t/6} + 12\left[-6te^{-\frac{t}{6}} + \int_0^{\infty} 6\ \ e^{-\frac{t}{6}}\right]\right] dt =$$

$$\frac{8}{3}\left[-6t^2e^{-t/6} + 12\left[-6te^{-\frac{t}{6}} - 36e^{-\frac{t}{6}}\right]\right] dt =$$

$$\frac{8}{3}\left[-6t^2e^{-t/6} - 72te^{-\frac{t}{6}} - 432e^{-\frac{t}{6}}\right] \text{ for t = 0 to } \infty =$$

$$\frac{8}{3} \times 432 = 1152.$$

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Discussion: wizardofvegas.com/forum/questions-and-answers/math/34502-easy-math-puzzles/162/#post800246