

Question

On a windless day, Charlie takes his canoe to the river and paddles up upstream. He always paddles at the same rate (in other words if we ignore the wind and current, he would always go the same speed). A mile after launching, his hat falls in the river. Ten minutes after that, he realizes his hat is missing and immediately makes a u-turn to catch up to it downstream. Charlie catches up to his hat at the same place he launched.

How fast is the current?

Scroll down for the answer.

Answer

The answer is 3 miles per hour.

Scroll down for the solution.

Hard Solution

Let's call:

c = rate of current.

p = rate Charlie can paddle in still water.

d = distance Charlie paddles upstream after his hat falls off.

t = time hat travels downstream.

Recall that distance = rate * time.

After Charlie's hat falls off, it travels downstream for a mile for time t . This can be expressed as:

$$1 = c \times t$$

After Charlie's hat falls off, Charlie continues paddling upstream a distance of d for 10 minutes. This can be expressed as:

$$d = (p-c) \times (1/6)$$

After Charlie turns around, he travels a distance of $1+d$ at a rate of $c+p$. He paddles 10 minutes less than it takes the hat to float. This can be expressed as:

$$1+d = (p+c) \times (t-1/6)$$

From $1=c \times t$, we know $t=1/c$. So, let's rewrite that as:

$$1+d = (p+c) \times (1/c - 1/6)$$

$$1+d = (p+c) \times (6-c)/6c$$

Recall $d = (p-c)/6$. Let's substitute that for d in the equation above:

$$1 + (p-c)/6 = (p+c) \times (6-c)/6c$$

$$(6+p-c)/6 = (p+c) \times (6-c)/6c$$

$$6c * (6+p-c) = 6 * (p+c) \times (6-c)$$

$$36c + 6cp - 6c^2 = 6*(6p - cp + 6c - c^2)$$

$$36c + 6cp - 6c^2 = 36p - 6cp + 36c - 6c^2$$

$$6cp - 6c^2 = 36p - 6cp - 6c^2$$

$$6cp = 36p - 6cp$$

$$12cp = 36p$$

$$c = 3$$

So, there you go, the speed of the current is 3 miles per hour.

Easy Solution

Let's say Charlie was in still water. If he traveled north for any period of time and then traveled south for the same period of time, he would end up in the same place. So, his energy cancels itself out.

The same thing happens in moving water. If he travels upstream and downstream the same period of time, then he would end up in the same spot as if he sat in the canoe doing nothing.

This is the case here. After Charlie's hat falls in the water, he paddles upstream for a while and then downstream. He catches up with his floating hat where he started. This suggests he paddled upstream, after his hat fell off, the same period of time he paddled downstream. So, he paddles without a hat, both ways, for $10+10 = 20$ minutes.

The hat must also float for that same 20 minutes. Since it floated for a mile, we can use $\text{distance} = \text{rate} * \text{time}$ so solve for the speed of the current:

$$1 = \text{rate} \times (1/3)$$

Rate = 3 miles per hour.