Q: What is the expected return of the Replay side bet in craps. The pay table is as follows (win on a "to one" basis):

EVENT	PAYS
Point of 4 or 10 achieved four or more times	1000
Point of 5 or 9 achieved four or more times	500
Point of 4 or 10 achieved three times	120
Point of 6 or 8 achieved four or more times	100
Point of 5 or 9 achieved three times	95
Point of 6 or 8 achieved three times	70

A: The key to solving this problem is that the answer is the same whether there is one significant discrete event at a time (any point being won or lost) or they happen at a point in time, with random periods of no events in between. Either way, there is a particular order of events that determines the outcome of the bet.

The probability of any significant event, given that one has happened is as follows:

Point of 4 or 10 win = 1/24.

Point of 5 or 9 win = 1/15.

Point of 6 or 8 win = 25/264

Any seven-out = 98/165

We shall use integral calculus to solve for each event. In each case, the mean time between significant events is 1. Time since the bet started is x. Any given probability is the sum from 0 to infinity that the first seven-out to occur happened at exactly that moment with the needed winning conditions happening before.

To do the integration, I recommend the calculator at www.integral-calculator.com. You can find the integrals in text form at the end.

a = Probability point of 4 or 10, but not both, achieved at least four times =

2 * (1-pr(point of 4 or 10 achieved 0 to 3 times) * pr(point of 4 or 10 achieved 0 to 3 times) * seven-out achieved 0 times * probability of seven-out, given a point made or lost. =

$$\int_0^\infty 2 \times (1 - e^{-\frac{x}{24}}(1 + \frac{x}{24} + \frac{\left(\frac{x}{24}\right)^2}{2!} + \frac{\left(\frac{x}{24}\right)^3}{3!})) \times (e^{-\frac{x}{24}}(1 + \frac{x}{24} + \frac{\left(\frac{x}{24}\right)^2}{2!} + \frac{\left(\frac{x}{24}\right)^3}{3!})) \times e^{-\frac{98x}{165}} \times \frac{98}{165}\, dx =$$

65201685378467494355681875/1766853353929146219986463522783

= apx. 3.6902714780188938 $\times 10^{-5}$

b = Probability point of both 4 and 10 achieved at least four times =

(1-pr(point of 4 or 10 achieved 0 to 3 times) * pr(point of 4 or 10 achieved 0 to 3 times) * seven-out achieved 0 times * probability of seven-out, given a point made or lost. =

$$\int_0^\infty (1 - e^{-\frac{x}{24}}(1 + \frac{x}{24} + \frac{\left(\frac{x}{24}\right)^2}{2!} + \frac{\left(\frac{x}{24}\right)^3}{3!}))^2 \times e^{-\frac{98x}{165}} \times \frac{98}{165}\, dx$$

= 56262929147846029296875/3533706707858292439972927045566 =

apx. 1.5921788025794 $\times 10^{-8}$

Probability point of 4 and/or 10 achieved at least four times = a + b = 3.6918636568215 $\times 10^{-5}$

c = Probability point of 5 or 9, but not both, achieved at least four times, and 4 and 10 both achieved 3 or less times. =

2 * (1-pr(point of 5 or 9 achieved 0 to 3 times) * pr(point of 5 or 9 achieved 0 to 3 times) * pr(4 or 10 achieved 0 to 3 times)^2 * seven-out achieved 0 times * probability of seven-out, given a point made or lost. =

$$\int_0^\infty 2 \times (1 - e^{-\frac{x}{15}} (1 + \frac{x}{15} + \frac{\left(\frac{x}{15}\right)^2}{2!} + \frac{\left(\frac{x}{15}\right)^3}{3!})) \times (e^{-\frac{x}{15}} (1 + \frac{x}{15} + \frac{\left(\frac{x}{15}\right)^2}{2!} + \frac{\left(\frac{x}{15}\right)^3}{3!})) \times (e^{-\frac{x}{24}} (1 + \frac{x}{24} + \frac{\left(\frac{x}{24}\right)^2}{2!} + \frac{\left(\frac{x}{24}\right)^3}{3!}))^2 \times e^{-\frac{98x}{165}} \times \frac{98}{165} \, dx =$$

632620488252916242831069804557124990244391362510561792/3066366468516034093381319822588282963792443265015898546875 = apx. 0.0002063094854279021

d = Probability point of both 5 or 9 achieved at least four times, and 4 and 10 both achieved 3 or less times. =

(1-pr(point of 5 or 9 achieved 0 to 3 times)^2 * pr(4 or 10 achieved 0 to 3 times)^2 * seven-out achieved 0 times * probability of seven-out, given a point made or lost. =

$$\int_0^\infty (1 - e^{-\frac{x}{15}} (1 + \frac{x}{15} + \frac{\left(\frac{x}{15}\right)^2}{2!} + \frac{\left(\frac{x}{15}\right)^3}{3!}))^2 \times (e^{-\frac{x}{24}} (1 + \frac{x}{24} + \frac{\left(\frac{x}{24}\right)^2}{2!} + \frac{\left(\frac{x}{24}\right)^3}{3!}))^2 \times e^{-\frac{98}{165}} \times \frac{98}{165} \, dx =$$

4432779172318394681215006889833826206938632308352870259873720186109952/10933940823420496140413926170733638530811352664699776845348274954672408453125 = apx. $4.054145933205879 \times 10^{-7}$

Prob 5 and/or 9 4+ times and no higher win = c + d = 0.000206714900021223.

e = Probability point of 4 or 10, but not both, achieved exactly three times, and 5 or 9 both achieved 3 or less times =

2 * pr(point of 4 or 10 achieved 3 times) * pr(point of 4 or 10 achieved 2 or less times) * pr(point of 5 or 9 achieved 0 to 3 times)^2 * seven-out achieved 0 times * probability of seven-out, given a point made or lost. =

$$\int_0^\infty 2 \times e^{-\frac{x}{24}} \times \frac{\left(\frac{x}{24}\right)^3}{3!} \times (e^{-\frac{x}{24}}(1+\frac{x}{24}+\frac{\left(\frac{x}{24}\right)^2}{2!})) \times (e^{-\frac{x}{15}}(1+\frac{x}{15}+\frac{\left(\frac{x}{15}\right)^2}{2!}+\frac{\left(\frac{x}{15}\right)^3}{3!}))^2 \times e^{-\frac{98x}{165}} \times \frac{98}{165}\, dx =$$

= 18371053133887106656384997/35190493577512864651856265625

= apx. 0.0005220459068987433

f = Probability point of 4 and 10 achieved exactly three times, and 5 or 9 both achieved 3 or less times =

pr(point of 4 or 10 achieved 3 times)^2 * pr(point of 5 or 9 achieved 0 to 3 times)^2 * seven-out achieved 0 times * probability of seven-out, given a point made or lost. =

$$\int_0^\infty (e^{-\frac{x}{24}} \times \frac{\left(\frac{x}{24}\right)^3}{3!})^2 \times (e^{-\frac{x}{15}}(1+\frac{x}{15}+\frac{\left(\frac{x}{15}\right)^2}{2!}+\frac{\left(\frac{x}{15}\right)^3}{3!}))^2 \times e^{-\frac{98}{165}} \times \frac{98}{165}\, dx =$$

1407672312689881759266669/1506153125117550607099448168750

= apx. 9.346143424693404 * 10^-7

Probability of 4 and/or 10 achieved exactly three times and no higher win = e + f = 0.00052298052124121200.

g = Probability point of 6 or 8, but not both, achieved 4 or more times, points of 5 and 9 both achieved three or less times, and points of 4 and 10 both achieved two or less times =

$$\int_0^\infty 2 \times (1 - e^{-\frac{25x}{264}}(1 + \frac{25x}{264} + \frac{\left(\frac{25x}{264}\right)^2}{2!} + \frac{\left(\frac{25x}{264}\right)^3}{3!})) \times (e^{-\frac{25}{264}}(1 + \frac{25x}{264} + \frac{\left(\frac{25x}{264}\right)^2}{2!} + \frac{\left(\frac{25}{264}\right)^3}{3!})) \times (e^{-\frac{x}{15}}(1 + \frac{x}{15} + \frac{\left(\frac{x}{15}\right)^2}{2!} + \frac{\left(\frac{x}{15}\right)^3}{3!}))^2 \times (e^{-\frac{x}{24}}(1 + \frac{x}{24} + \frac{\left(\frac{x}{24}\right)^2}{2!}))^2 \times e^{-\frac{98x}{165}} \times \frac{98}{165}\, dx =$$

464463634046049935049755642596110138619810984882625/93080375389750
11742105498920371662196076747088233234432 =
apx. 0.0006945033909648271

h = Probability point of both 6 and 8 achieved 4 or more times, points of 5 and 9 both achieved three or less times, and points of 4 and 10 both achieved two or less times =

$$\int_0^\infty (1 - e^{-\frac{25x}{264}}(1 + \frac{25x}{264} + \frac{\left(\frac{25x}{264}\right)^2}{2!} + \frac{\left(\frac{25}{264}\right)^3}{3!}))^2 \times (e^{-\frac{x}{15}}(1 + \frac{x}{15} + \frac{\left(\frac{x}{15}\right)^2}{2!} + \frac{\left(\frac{x}{15}\right)^3}{3!}))^2 \times (e^{-\frac{x}{24}}(1 + \frac{x}{24} + \frac{\left(\frac{x}{24}\right)^2}{2!}))^2 \times e^{-\frac{98x}{165}} \times \frac{98}{165}\, dx =$$

14522454098331213476368862097692665221653133074880831145217081273
92578125/391840819719834080066936357191044966832527642531881569596
370273664050032279552 = apx. 3.706212667867212 * 10^-6

Probability point of 6 and/or 8 achieved at least four times and no higher win = g + h = 0.00069820960363269400 i = Probability 5 or 9, but not both, three times, probability 5 or 9 achieved two or less times, both 4 and 10 achieved two or less, and both 6 and 8 three of less times =

2 * pr(5 or 9 exactly three times) * pr(5 or 9 two or less times) * pr(4 or 10 achieved two or less times)^2 + pr(6 or 8 three of less times)^2 =

$$\int_0^\infty 2 \times e^{-\frac{x}{15}} \times \frac{\left(\frac{x}{15}\right)^3}{3!} \times \left(e^{-\frac{x}{15}} \left(1 + \frac{x}{15} + \frac{\left(\frac{x}{15}\right)^2}{2!}\right)\right) \times \left(e^{-\frac{x}{24}} \left(1 + \frac{x}{24} + \frac{\left(\frac{x}{24}\right)^2}{2!}\right)^2 \times \left(e^{-\frac{25x}{264}}\left(1 + \frac{25x}{264} + \frac{\left(\frac{25x}{264}\right)^2}{2!} + \frac{\left(\frac{25x}{264}\right)^3}{3!}\right)\right)^2 \times e^{-\frac{98x}{165}} \times \frac{98}{165}\, dx =$$

364633390501256729657/203612782134774988800000 =

apx. 0.001790817780093488

j = Probability 5 and 9 three times each, probability 5 or 9 achieved two or less times, both 4 and 10 achieved two or less, and both 6 and 8 three of less times =

pr(5 or 9 exactly three times)^2 * pr(5 or 9 two or less times) * pr(4 or 10 achieved two or less times)^2 + pr(6 or 8 three of less times)^2 =

$$\int_0^\infty \left(e^{-\frac{x}{15}} \times \frac{\left(\frac{x}{15}\right)^3}{3!}\right)^2 \times \left(e^{-\frac{x}{24}} \left(1 + \frac{x}{24} + \frac{\left(\frac{x}{24}\right)^2}{2!}\right)^2 \times \left(e^{-\frac{25x}{264}}\left(1 + \frac{25x}{264} + \frac{\left(\frac{25}{264}\right)^2}{2!} + \frac{\left(\frac{25x}{264}\right)^3}{3!}\right)\right)^2 \times e^{-\frac{98}{165}} \times \frac{98}{165}\, dx =$$

379628902142645911/41648069073022156800000 =

apx. 9.11516213337615 * 10^-6

Probability 5 and/or 9 achieved exactly three times, both 4 and 10 achieved two or less, and both 6 and 8 three of less times = i + j = 0.00179993294222686000.

k = probability 6 or 8, but not both, achieved three times, and all other points 2 or less times each. =

2*pr(6 or 8 achieved exactly three times) * pr(6 or 8 achieved two or less times) * pr(5 or 9 achieved two or less times)^2 * pr(4 or 10 achieved two or less times)^2

$$\int_0^\infty 2 \times e^{-\frac{25x}{264}} \times \frac{\left(\frac{25x}{264}\right)^3}{3!} \times (e^{-\frac{25x}{264}}(1+\frac{25x}{264}+\frac{\left(\frac{25x}{264}\right)^2}{2!})) \times (e^{-\frac{x}{15}}(1+\frac{x}{15}+\frac{\left(\frac{x}{15}\right)^2}{2!}))^2 \times (e^{-\frac{x}{24}}(1+\frac{x}{24}+\frac{\left(\frac{x}{24}\right)^2}{2!}))^2 \times e^{-\frac{98}{165}} \times \frac{98}{165}\, dx =$$

1864414243904975/438761550728134656 = apx. 0.004249265326031731

l = probability 6 and 8, achieved three times, and all other points 2 or less times each. =

pr(6 or 8 achieved exactly three times)^2 * pr(5 or 9 achieved two or less times)^2 * pr(4 or 10 achieved two or less times)^2

$$\int_0^\infty (e^{-\frac{25x}{264}} \times \frac{\left(\frac{25}{264}\right)^3}{3!})^2 \times (e^{-\frac{x}{15}}(1+\frac{x}{15}+\frac{\left(\frac{x}{15}\right)^2}{2!}))^2 \times (e^{-\frac{x}{24}}(1+\frac{x}{24}+\frac{\left(\frac{x}{24}\right)^2}{2!}))^2 \times e^{-\frac{98}{165}} \times \frac{98}{165}\, dx =$$

15293145095703125/347499148176682647552 = apx. 4.40091585143324*10^-5

probability 6 and/or 8 achieved three times, and all other points 2 or less times each. = k + l = 0.004293274484546060000

Probability loser =
pr(6 or 8 two or less times)^2 * pr(5 or 9 two or less times)^2 * pr(4 or 10 two or less times)^2 =

$$\int_0^{\infty} \left(e^{-\frac{25x}{264}}\left(1+\frac{25}{264}+\frac{\left(\frac{25x}{264}\right)^2}{2!}\right)\right)^2 \times \left(e^{-\frac{x}{15}}\left(1+\frac{x}{15}+\frac{\left(\frac{x}{15}\right)^2}{2!}\right)\right)^2 \times \left(e^{-\frac{x}{24}}\left(1+\frac{x}{24}+\frac{\left(\frac{x}{24}\right)^2}{2!}\right)\right)^2 \times e^{-\frac{98x}{165}} \times \frac{98}{165}\, dx =$$

127269622523676773/128238855782400000 = apx. 0.9924419689117637

Summary

The following table summarizes the probability and contribution to the return of all possible outcomes. The lower right cell shows a house edge of 24.81%.

EVENT	PAYS	PROBABILITY	RETURN
Point of 4 or 10 achieved four or more times	1000	0.00003691864	0.03691863657
Point of 5 or 9 achieved four or more times	500	0.00020671490	0.10335745001
Point of 4 or 10 achieved three times	120	0.00052298052	0.06275766255
Point of 6 or 8 achieved four or more times	100	0.00069820960	0.06982096036
Point of 5 or 9 achieved three times	95	0.00179993294	0.17099362951
Point of 6 or 8 achieved three times	70	0.00429327448	0.30052921392
Loser	-1	0.99244196891	-0.99244196891
Total		1.00000000000	-0.24806441599

Integrals in text form:

a = 2*(1-exp(-x/24)*(1+(x/24)+(x/24)^2/2+(x/24)^3/6))*(exp(-
x/24)*(1+(x/24)+(x/24)^2/2+(x/24)^3/6))*exp(-98*x/165)*(98/165)

b = (98/165)*(1-exp(-x/24)*(1+(x/24)+(x/24)^2/2+(x/24)^3/6))^2*exp(-98*x/165)

c = 2*(1-exp(-x/15)*(1+(x/15)+(x/15)^2/2+(x/15)^3/6))* (exp(-
x/15)*(1+(x/15)+(x/15)^2/2+(x/15)^3/6))*(exp(-
x/24)*(1+(x/24)+(x/24)^2/2+(x/24)^3/6))^2*exp(-98*x/165)*(98/165)

d = (98/165)*(1-exp(-x/15)*(1+(x/15)+(x/15)^2/2+(x/15)^3/6))^2*(exp(-
x/24)*(1+(x/24)+(x/24)^2/2+(x/24)^3/6))^2*exp(-98*x/165)

e = 2*(exp(-x/24)*(x/24)^3/6)*(exp(-x/24)*(1+(x/24)+(x/24)^2/2)*(exp(-
x/15)*(1+(x/15)+(x/15)^2/2+(x/15)^3/6))^2*exp(-98*x/165)*(98/165)

f = (exp(-x/24)*(x/24)^3/6)^2*(exp(-
x/15)*(1+(x/15)+(x/15)^2/2+(x/15)^3/6))^2*exp(-98*x/165)*(98/165)

g = 2*(1-exp(-x*25/264)*(1+x*(25/264)+(x*25/264)^2/2+(x*25/264)^3/6))*(exp(-
x*25/264)*(1+x*(25/264)+(x*25/264)^2/2+(x*25/264)^3/6))*(exp(-
x/15)*(1+(x/15)+(x/15)^2/2+(x/15)^3/6))^2*(exp(-
x/24)*(1+(x/24)+(x/24)^2/2))^2*exp(-98*x/165)*(98/165)

h = (98/165)*(1-exp(-
x*25/264)*(1+x*(25/264)+(x*25/264)^2/2+(x*25/264)^3/6))^2*(exp(-
x/15)*(1+(x/15)+(x/15)^2/2+(x/15)^3/6))^2*(exp(-
x/24)*(1+(x/24)+(x/24)^2/2))^2*exp(-98*x/165)

i = 2*exp(-x/15)*(x/15)^3/6*(exp(-x/15)*(1+(x/15)+(x/15)^2/2)*(exp(-
x/24)*(1+(x/24)+(x/24)^2/2))^2*(exp(-
x*25/264)*(1+x*(25/264)+(x*25/264)^2/2+(x*25/264)^3/6))^2*exp(-
98*x/165)*(98/165)

j = (exp(-x/15)*(x/15)^3/6)^2*(exp(-x/24)*(1+(x/24)+(x/24)^2/2))^2*(exp(-x*25/264)*(1+x*(25/264)+(x*25/264)^2/2+(x*25/264)^3/6))^2*exp(-98*x/165)*(98/165)

k = 2*exp(-x*25/264)*(x*25/264)^3/6*(exp(-x*25/264)*(1+(x*25/264)+(x*25/264)^2/2))*(exp(-x/24)*(1+(x/24)+(x/24)^2/2))^2*(exp(-x/15)*(1+(x/15)+(x/15)^2/2))^2*exp(-98*x/165)*(98/165)

l = (exp(-x*25/264)*(x*25/264)^3/6)^2*(exp(-x/24)*(1+(x/24)+(x/24)^2/2))^2*(exp(-x/15)*(1+(x/15)+(x/15)^2/2))^2*exp(-98*x/165)*(98/165)

Loser = (exp(-x*25/264)*(1+(x*25/264)+(x*25/264)^2/2))^2*(exp(-x/24)*(1+(x/24)+(x/24)^2/2))^2*(exp(-x/15)*(1+(x/15)+(x/15)^2/2))^2*exp(-98*x/165)*(98/165)

Recommended integral calculator: www.integral-calculator.com