Question: What is the average number of rolls of two dice to achieve a total of 12 three consecutive times?

Answer: 47988

Solution:

Let:

x = expected number of additional throws from starting point or after any roll that isn't a 12.

y = expected number of additional throws after a single 12 in the previous throw.

z = expected number of additional throws after two 12's the previous two throws.

This turns into a Markov chain problem, as follows:

(1) x = 1 + (35/36)*x + (1/36)*y
(2) y = 1 + (35/36)*x + (1/36)*z
(3) z = 1 + (35/36)*x

Substitute the value of z from equation (3) into z in equation (2):

y = 1 + (35/36)*x + (1/36)*(1 + (35/36))*x

Multiply both side by 36:

36y = 36 + 35x + (1+(35/36))*x

Multiply again by 36:

1296y = 1296 + 1260x + (36+35x)

1296y = 1295x + 1332

(4) y = (1332 + 1295x)/1296

Substitute the value of y in equation (4) into y in equation (1):

(1) x = 1 + (35/36)*x + (1/36)* (1332 + 1295x)/1296

Multiply both sides by 1296:

1296x = 1296 + 1260x + (1/36)*(1332+1295x)

Multiply both sides by 36:

46656x = 46656 + 45360x + 1332 + 1295x

x = 47988

Here are two expressions for x:

$$x = 36^3 + 36^2 + 36^1$$

$$x = \frac{36^4 - 1}{36 - 1} - 1$$